**Adaptive Control - Autotuning**

**Structure of presentation:**
- Relay feedback autotuning – outline
- Relay feedback autotuning – details
- How close is the estimate of the ultimate gain and period to the actual ultimate gain and period?
- The effect of noise
- Determination of the gain and phase margins using relay autotuning
- Static load disturbance
- Commercial autotuning controller
- Question and Answer

---

**1. Relay feedback autotuning - outline**

**Reminder of ultimate cycle tuning**

1. Place the controller in proportional mode only (i.e. set $T_i$ to a maximum and $T_d$ to a minimum).
2. Increase $K_c$ until the closed loop system output goes “marginally stable”; record $K_c$ (calling it $K_u$, the *ultimate gain*), and the *ultimate period*, $T_u$.

**PI controller settings:**

- $K_c = 0.45K_u$, $T_i = 0.83T_u$

**Ideal PID controller settings:**

- $K_c = 0.6K_u$, $T_i = 0.5T_u$
- $T_d = 0.125T_u$


---

**Modified ultimate cycle method - relay auto-tuning**

- Åström and Hägglund (1984) have developed an attractive alternative to the ultimate cycle method.
- In the *relay auto-tuning* method, a simple experimental test is used to determine $K_u$ and $T_u$.
- For this test, the feedback controller is temporarily replaced by an on-off controller (or *relay*).
- After the control loop is closed, the controlled variable exhibits a sustained oscillation that is characteristic of on-off control.
- The operation of the relay auto-tuner includes a *dead band*; the dead band is used to avoid frequent switching caused by measurement noise.


---

The relay auto-tuning method has several important advantages over the ultimate cycle method:

1. Only a single experiment test is required instead of a trial-and-error procedure.
2. The amplitude of the process output can be restricted by adjusting relay amplitude $d$.
3. The process is not forced to a stability limit.
4. The experimental test is easily automated using commercial products.
2. Relay feedback autotuning - details

When a relay is switched in, a sustained oscillation at $c$ is observed:

![Diagram showing relay feedback autotuning](image)

This oscillation is almost sinusoidal, depending on the filtering properties of $G_p(s)$. A comparison can be drawn to the oscillation obtained from the ultimate cycle experiment:

![Diagram showing ultimate cycle experiment](image)

Relay feedback autotuning - details

Knowledge of $K_u$ and $T_u$ allows the controller to be retuned using simple tuning methods (such as those of Ziegler and Nichols).

The relay autotuner will supply an approximate estimate of $T_u$ (labelled $\hat{T}_u$). An equivalent approximate $K_u$ (labelled $\hat{K}_u$) may be deduced from the relay autotuner using describing function analysis:

![Diagram showing describing function analysis](image)

From the tables of describing functions, $N(A) = \frac{4d}{\pi A}$, $A = \frac{1}{2}$ half-peak amplitude of limit cycle output.

Relay feedback autotuning - details

The limit cycle is defined when $G_p(j\omega)$ intersects $-\frac{1}{N(A)}$.

![Diagram showing limit cycle](image)

The relay autotuner can be considered equivalent to:
Relay feedback autotuning - details

An oscillatory output at $c$ exists when $G_p(j\omega) = \frac{1}{\kappa_u} = e^{-180^\circ}$

Now $G_p(j\omega) = \frac{-\pi A}{4\pi} \left( = \frac{\pi A}{4\pi} \right)$

Therefore, $\frac{1}{\kappa_u} = \frac{\pi A}{4\pi}$

$\therefore \kappa_u = \frac{4\pi}{\pi A}$

3. How close is the estimate of the ultimate gain and period to the actual ultimate gain and period?

Example: A process transfer function is $G_p(s) = \frac{1.84 - 3.49s}{1 + 7.67s}$

Determine, using SIMULINK, $\kappa_u$ and $\tau_u$ and compare with the values of $\kappa_u$, $\tau_u$ calculated analytically.

Analytical calculation:

Simulation of relay autotuner

\[ G_p(j\omega) = \frac{\kappa_u \cdot 1.84 - 3.49j\omega}{1 + 7.67j\omega} \]

\[ |G_p(j\omega)| = \frac{|1.84 - 3.49j\omega|}{|1 + 7.67j\omega|} = 1 \quad \text{---- (A)} \]

\[ \arg G_p(j\omega) = -\tan^{-1}(7.67\omega) - 3.49\omega = -\pi \text{ radians.} \quad \text{---- (B)} \]

Determine $\omega_u$ from equation (B) [by iteration] and then solve equation (A) to determine $\kappa_u$.

$\omega_u = 0.5 \text{ radians}:
\LHS = -\tan^{-1}(3.84) - 1.74 = -3.05 \text{ radians}$

$\omega_u = 0.55 \text{ radians}:
\LHS = -\tan^{-1}(4.22) - 1.91 = -3.24 \text{ radians}$

$\omega_u = 0.52 \text{ radians}:
\LHS = -\tan^{-1}(3.99) - 1.80 = -3.13 \text{ radians}$

[close enough]

From equation (A), $\kappa_u = \sqrt{1 + (7.67 \cdot 0.52)^2} = 2.23$

$\tau_u = \frac{2\pi}{\omega_u} = \frac{2\pi}{0.52} = 12.08 \text{s}$
Estimate of ultimate gain and ultimate period from simulation

- Peak to peak amplitude (= 2A) = 1.34 i.e. A = 0.67;
- \( d = 1 \) (in the simulation);
- Therefore, \( \frac{V_u}{kd} = 1.90 \).
- From the simulated output, the estimate of the ultimate period = 14 seconds.

Overall: 
\( V_u = 1.90 \); \( V_u = 2.23 \) [\( \% \) err = 15%] \( T_u = 14.5 \); \( T_u = 12.15 \). [\( \% \) err = 16%]

Such large \% errors are related to the sinusoidal nature of the limit cycle output.

Example 2

Simulation of relay autotuner

A process transfer function is
\[ G_p(s) = \frac{2e^s}{(1+1.5s+3s^2)(1+4s)} \]

Determine, using SIMULINK, \( V_u \) and \( T_u \) and compare with the values of \( V_u \) and \( T_u \) calculated analytically.

Analytical calculation:
\[ G_p(s) \cdot G_e(s) = \frac{V_u \cdot 2e^{1.5s}}{(1+1.5s)(1+2s)(1+4s)} \]

\( G_p(s) \cdot G_e(s) = \frac{V_u \cdot 2e^{1.5s}}{(1+1.5s)(1+2s)(1+4s)} \)

Simulation of relay autotuner
Estimate of ultimate gain and ultimate period from simulation

• Peak to peak amplitude (= 2A) = 1.13 i.e. A = 0.57;
• d = 1 (in the simulation);
• Therefore, \[ \frac{\nu_u}{\pi A} = 2.25. \]
• From the simulated output, the estimate of the ultimate period = 14.3 seconds.

Overall:

\[ \frac{\nu_u = 2.25 \text{ sec} \text{ for } \nu_u = 2.32 \text{ sec}}{\text{error } 3\%} \]

\[ \frac{\nu_u = 13.15 \text{ sec} \text{ for } \nu_u = 13.15 \text{ sec}}{\text{error } 9\%} \]

… More sinusoidal limit cycle output -> less percentage error.

4. The effect of noise

The presence of noise can cause difficulties in measuring the amplitude and period of the limit cycle output. In a simulation (noise amplitude: max. = 0.2; min. = -0.2):

One possibility: Use hysteresis on the relay

Generally, the hysteresis width is made larger than the maximum noise level; if the maximum noise amplitude is ±0.2, set up hysteresis on the relay to be ±0.25, say.

However, introducing hysteresis changes the amplitude and frequency of the controlled variable …
Measuring amplitude and period

With or without hysteresis, the challenge is to measure accurately the amplitude and period of the controlled variable. Without hysteresis: Estimated peak-peak amplitude = 0.96, estimated period = 14.6 s. With hysteresis: Estimated peak-peak amplitude = 1.53, estimated period = 16.5 s.

Estimating ultimate gain

If hysteresis is absent, the ultimate gain may be estimated as 2.65. The following table summarises the results:

<table>
<thead>
<tr>
<th></th>
<th>Actual</th>
<th>Relay, no noise</th>
<th>Relay, noise present</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ultimate gain</td>
<td>2.32</td>
<td>2.25 (-3%)</td>
<td>2.65 (+14%)</td>
</tr>
<tr>
<td>Ultimate period</td>
<td>13.1</td>
<td>14.3 (+9%)</td>
<td>14.6 (+11%)</td>
</tr>
</tbody>
</table>

If hysteresis is present, N(A) [from describing function analysis] changes … see table of describing functions previously. Thus, the ultimate gain is estimated by a different formula. This is left as an exercise.

5. Determination of the gain and phase margins using relay autotuning

Reminder: Gain and phase margin

Gain margin - relay autotuning

A relay autotuner may be used to estimate the gain margin of compensated systems. The method is set up as follows:

\[ G_p(s) = \frac{2e^{-s}}{(1+1.5s)(1+3s)(1+4s)} \]
Gain margin – relay autotuning

is approximated by:

\[ G_m(s) = \frac{2(1 + 2.25s)e^{-\theta}}{1 + 8.5s + 22.5s^2 + 18s} \]

is modelled as: \( G_m(s) = \frac{1.95e^{-1.87s}}{1 + 6.69s} \)

The PI controller was designed, using a standard method, to achieve the specifications of ±2% settling time of 25 seconds and a phase margin of 45°.

We will use MATLAB to determine the gain margin of this compensated system, and compare the result to the estimate of the gain margin obtained from the relay test.

Example: A process, given by

\[ G_p(s) = \frac{1}{s^2 + 5s + 18} \]

is modelled as:

\[ G_m(s) = \frac{1.95e^{-1.87s}}{1 + 6.69s} \]

\[ G_c(s) = 1.36 \left(1 + \frac{1}{5.91s}\right) \]

The PI controller was designed, using a standard method, to achieve the specifications of ±2% settling time of 25 seconds and a phase margin of 45°.

We will use MATLAB to determine the gain margin of this compensated system, and compare the result to the estimate of the gain margin obtained from the relay test.

Gain margin – relay autotuning

Now, it may be shown that \( \dot{\omega} \) is the frequency where \( G_0(j\omega)G_r(j\omega) \) has a phase lag of 180°.

Reason: If this is true, \( G_0(j\omega)G_r(j\omega) = \frac{1}{-180°} \)

Substituting into equation (A):

\[ \frac{k_m}{k_{I}} = \frac{\omega - \omega_c}{\omega - \omega_m} = 1 - 180° \]

Therefore, \( \frac{k_m}{k_{I}} = \frac{\pi}{4\Delta} \)

Gain margin = \( \frac{1}{b} \cdot \text{Gain margin} = \frac{4 + \pi}{4\Delta} \)

Gain margin – relay autotuning

% Name of file: gain_margin.m
% % Set up transfer function for process (excluding delay)
% num = [4.5 2];
den = [18 22.5 8.5 1];
% % Set up transfer function for controller
% numc = [8.04 1.36];
denc = [5.91 0];
% % Now set up overall open loop transfer function
% [num,den] = series(numc,den,n,nc,nc);
sys = tf(num,den,'d',1);
% [Gm,Pm,wcg] = margin(num,phase,w)

Gm = 2.0350
Pm = 30.9096
wcg = 0.7260
wcp = 0.4469
Gain margin – relay autotuning

- $d = 1$ (in the simulation);
- $2A = 2.34$ i.e. $A = 1.17$;
- Therefore, gain margin $= \frac{4(1) + \pi(1.17)}{\pi(1.17)} = 2.09$ (estimate)

- Gain margin (from MATLAB simulation) $= 2.035$
- Error in gain margin estimate = $+3\%$.

Phase margin – relay autotuning

A relay autotuner may be used to estimate the gain margin of compensated systems. The method is set up as follows:

The transfer function of the “system” in series with the relay is:

$G_{CL} = \frac{1}{s} \frac{G_C G_P}{1 + G_C G_P} \left[1 - \frac{1}{G_C G_P}\right] = \frac{1}{s} \frac{G_C G_P - 1}{1 + G_C G_P}$
Example: A process, given by

\[ G_p(s) = \frac{2(1 + 2.25s)e^{-s}}{1 + 8.5s + 22.5s^2 + 18s^3} \]

is modelled as: \( G_m(s) = \frac{1.95e^{-1.87s}}{1 + 6.69s} \)

The PI controller was designed, using a standard method, to achieve the specifications of \( \pm 2\% \) settling time of 25 seconds and a phase margin of 45°.

We will use MATLAB to determine the phase margin of this compensated system, and compare the result to the estimate of the phase margin obtained from the relay test.

\[ \text{Phase margin} = 31^\circ \] (see slide 28).
Phase margin - relay autotuning

- From plot, phase lag = 6.2 seconds
- Period of waveform = 14 seconds
- Phase lag (in degrees) = (6.2/14.5)x360 = 154 degrees
- Estimated phase margin = 180 – 154 = 26 degrees.
- Phase margin (from MATLAB simulation) = 30.9 degrees
- Error in phase margin estimate = -16%.

6. Static load disturbance

Static load disturbances during the relay tuning experiment introduce errors in the estimates of the ultimate gain and ultimate period. This section shows how an automatic bias can be introduced to overcome the problem.

We consider the effects of a static load disturbance when the autotuner is a relay without hysteresis:


Static load disturbance

A typical static load disturbance occurs in a heating and ventilation system if environmental conditions change:

Thus, if outside temperature increases (for example), the heater does not need to be on as long to maintain the desired temperature. The ultimate period changes (as does the ultimate gain), which has a knock-on effect on the subsequent controller tuning.
The load disturbance may be determined, as follows:

The DC component of the manipulated variable (process input), $m_{dc}$, is

$$m_{dc} = \frac{r}{K_p} + \frac{1}{K_p} \int_0^{t_1+t_2} (y-r) \, dt$$

DC component with no load disturbance

DC component of the relay waveform

Now, $y_{dc} = K_p m_{dc}$

i.e.

$$K_p \left[ \frac{r}{K_p} + \frac{1}{K_p} \int_0^{t_1+t_2} (y-r) \, dt \right] = r + \frac{1}{(t_1 + t_2)} \int_0^{t_1+t_2} (y-r) \, dt$$

i.e.

$$K_p l + K_p \left( \frac{t_1-t_2}{t_1+t_2} \right) = \frac{1}{(t_1 + t_2)} \int_0^{t_1+t_2} (y-r) \, dt$$

To cancel the effect of the static load during the autotuning test, a bias, $u_b$, equal to the negative of the estimated load should be added to the relay output:

$$u_b = \left( \frac{t_1-t_2}{t_1+t_2} \right) d - \frac{1}{K_p (t_1 + t_2)} \int_0^{t_1+t_2} (y-r) \, dt$$

Note the estimated value of process gain used. This bias term may be automatically incorporated into an existing relay autotuner.
A process is given by: \( G_p(s) = \frac{e^{-s}}{(1 + s)^2} \)

When an autotuning relay is incorporated into the loop, the following response is determined (static load introduced after approximately 10 seconds, corrective bias term introduced after 22 seconds):

When a small static load disturbance (of 0.08) is introduced at approximately \( t = 10 \); the oscillations become asymmetrical for the next 12 seconds. The error that would result in the estimated ultimate gain and ultimate period is +14% and +21%, respectively.

At \( t = 22 \), a bias is applied based on an initial estimate of the process gain of 0.5 (noting that the actual process gain = 1). Exact symmetrical oscillations are not achieved; however, the asymmetry is slight, and the error that would result in the estimated ultimate gain and ultimate period is +2% and -2%, respectively.
7. Commercial autotuning controller

Example: Fisher DPR 900 Controller


---

Commercial autotuning controller

Some features:
1. If no process knowledge exists, autotuning is performed as follows:
   - The process is brought to a desired operating point, either by the operator in manual mode or by the controller in automatic mode
   - When the loop is stationary, the operator presses a tuning button.

   The PID controller is temporarily disconnected and the noise level is measured.

2. The PID controller parameters are then calculated from the ultimate gain and the ultimate period. Fast, medium or slow responses are available; for example, the medium tuning formula is $K_c = 0.35K_u$, $T_i = 1.13T_u$ and $T_d = 0.20T_u$.

   Typical performance:
   \[
   G_p = \frac{1}{(1+5s)(1+s)}
   \]
8. Question and Answer

A relay autotuning system is set up as follows:

(a) (i) Outline how relay feedback autotuning may be used to determine an approximate ultimate gain, \( K_u \), and approximate ultimate frequency, \( T_u \), of the closed loop system.
(ii) The relay autotuner is switched into the system. The resulting limit cycle output has a peak-to-peak amplitude of 1.22 and a period of 12.1 seconds. Determine \( K_u \) and \( T_u \).
(iii) Calculate exact values of ultimate gain, \( K_u \), and ultimate frequency, \( T_u \), when the proportional controller is connected to the process. Comment on the significance, or otherwise, of the differences between the approximate and exact values of ultimate gain and ultimate period.

(b) Detail how a relay feedback autotuner may be made robust to static load disturbances.
Answer

(b) Robust to static load disturbance

The ultimate period estimate changes in the presence of a static load disturbance. The load disturbance may, however, be determined. The DC component of the process input, \( \Delta u_c \), is

\[ \Delta u_c = \frac{r}{k_p} + \frac{t_{i+2}}{t_{i+2}}, \]

\( k_p = \) static gain of the process.

The DC component of the process output, \( y_{dc} \), is

\[ y_{dc} = \text{DC output without oscillation} \]

\[ = r + \frac{1}{t_{i+2}} \int_0^{t_{i+2}} (y_r - r) \, dt \]

New, \( y_{dc} = k_p, Y_{dc} = r + \frac{1}{t_{i+2}} \int_0^{t_{i+2}} (y_r - r) \, dt = k_p \left[ \frac{r}{k_p} + \frac{t_{i+1}}{t_{i+2}} \right] \]

\[ = k_p \frac{r}{k_p} + k_p \frac{t_{i+1}}{t_{i+2}} = \frac{1}{t_{i+2}} \int_0^{t_{i+2}} (y_r - r) \, dt \]

\[ \Rightarrow x = \frac{1}{t_{i+2}} \int_0^{t_{i+2}} (y_r - r) \, dt \]

To cancel the effect of the static load during the autotuning test, a bias equal to the negative of the estimated load should be added to the relay output.

Answer

(c) Closed loop transfer function

\[ G_{cl}(s) = \frac{G(s)P(s)}{1 + G(s)P(s)} \]

A limit cycle exists as a result of introducing the relay, \( \left| G_{cl}(j\omega) \right| = 180^\circ \). \( G(s)P(s) \) has a phase lag of 180°. Reason: if \( G(s)P(s) \) is

\[ G(s)P(s) = \frac{b}{1 + bs}, \]

then \( G(s)P(s) \) has a phase lag of 180°. Hence, \( G(s)P(s) \) may be approximated.

\[ G(s)P(s) = \frac{b}{1 + bs} \]

Then, \( G(s)P(s) \) may be approximated as

\[ \left| G(s)P(s) \right| = 1 \quad \text{for gain margin} (\frac{1}{b}) \]